Probability Homework #5

(Coverage: 4.2, 4.3, 4.4, 4.5)

1. Two fair dice are rolled and the absolute value of the difference of the outcomes is denoted by X. What are the possible values of X, and the probabilities associated with them?

2. F, the distribution function of a random variable X, is given by

$$F(t) = \begin{cases} 0 & t < -1 \\ (1/4)t + 1/4 & -1 \le t < 0 \\ 1/2 & 0 \le t < 1 \\ (1/12)t + 7/12 & 1 \le t < 2 \\ 1 & t \ge 2. \end{cases}$$

   Then, calculate the following quantities: $P(X < 1)$, $P(X = 1)$, $P(0 \le X < 1)$, $P(X > 1/2)$, $P(X = 3/2)$, and $P(1 < X \le 6)$.

3. Assume that 12% of people have traveled internationally. A company wants to hire a new employee who has traveled internationally. How many applicants do they need to interview to have a 60% chance that at least one of the applicants has traveled internationally??

4. In the experiment of rolling a balanced die twice, let X be the minimum of the two numbers obtained. Determine the probability mass function and the distribution function of X.

5. In a lottery every week, 2,000,000 tickets are sold for $1 apiece. If 4000 of these tickets pay off $30 each, 500 pay off $800 each, one ticket pays off $1,200,000, What is the expected value of the winning amount for a player with a single ticket?

6. If X is a random number selected from the first 10 positive integers, what is $E[X(11-X)]$?

7. Mr. Jones is about to purchase a business. There are two businesses available. The first has a daily expected profit of $150 with standard deviation $30, and the second has a daily expected profit of $150 with standard deviation $55. If Mr.

Jones is interested in a business with a steady income, which should he choose?

8. Find the variance and the standard deviation of a random variable X with distribution function

$$F(x) = \begin{cases} 0 & x < -3 \\ 3/8 & -3 \le x < 0 \\ 3/4 & 0 \le x < 6 \\ 1 & x \ge 6. \end{cases}$$

9. Suppose that X is a discrete random variable with $E[X]=1$ and $E[X(X-2)]=3$. Find $Var(-3X+7)=?$