Probability Homework #6

(Coverage: Chapter 5)

1. A graduate class consists of six students. What is the probability that exactly three of them are born either in April or in October?

2. A certain basketball player makes a successful foul shot with probability 0.45. Determine for what value of k, the probability of k baskets in 10 shots is maximum, and then find this maximum probability.

3. The simplest error detection scheme used in data communication is parity checking. Each character consists of a number of bits (a *bit* is the smallest unit of information and is either 1 or 0). In parity checking, a 1 or 0 is appended to the end of each character at the transmitter to make the total number of 1's even. The receiver checks the number of 1's in every character received, and if the result is odd it signals an error. Suppose that each bit is received correctly with probability 0.999, independently of other bits. What is the probability that a 7-bit character is received in error, but the error is not detected by the parity check?

4. Suppose that 2.5% of the population of a town are illegal immigrants. Find the probability using Poisson random variable approximation that, in a theater of this town with 80 random viewers, there are at least two illegal immigrants.

5. On average, there are three misprints in every 10 pages of a particular book. If every chapter of the book contains 35 pages, what is the probability (by using Poisson model approximation) that Chapters 1 and 5 have 10 misprints each?

6. Suppose that X is a Poisson random variable with $P(X = 1) = P(X = 3)$. Find $P(X = 5)$.

7. Suppose that 20% of a group of people have hazel eyes. What is the probability that the eighth passenger boarding a plane is the third one having hazel eyes? Assume that passengers boarding the plane form a randomly chosen group.

8. The probability is p that a randomly chosen light bulb is defective. We screw a bulb into a lamp and switch on the current. If the bulb works, we stop; otherwise, we try another and continue until a good bulb is found. What is the probability that at least n bulbs are required?

9. Suppose that 15% of the population of a town are senior citizens. Let X be the number of nonsenior citizens who enter a mall before the tenth senior citizen arrives. Find the probability mass function of X. Assume that each customer who enters the mall is a random person from the entire population.

10. To estimate the number of trout in a lake, we caught 50 trout, tagged and returned them. Later we caught 50 trout and found that four of them were tagged. From this experiment result, please estimate n, the total number of trout in the lake.

    Hint: Let P_n be the probability of four tagged trout among the 50 trout caught, if the total number of trout in the lake is n. Then, find the value of n that maximizes P_n. (Note: In statistics, this approach to estimate value of n is called the *maximum likelihood estimate* for the number of trout in lake. For this question, you may first find P_n, using the hypergeometric random variable. Then use the relation $P_n / P_{n+1} >= 1$ or $P_n / P_{n-1} <= 1$ to find the value of n that maximizes P_n.)