

Probability Homework#9
(Coverage: 8.1, 8.2, 8.3)

(1)

Let the joint probability mass function of discrete random variables X and Y be given by

$$p(x, y) = \begin{cases} c(x + y) & \text{if } x = 1, 2, 3, \quad y = 1, 2 \\ 0 & \text{otherwise.} \end{cases}$$

Determine (a) the value of the constant c , (b) the marginal probability mass functions of X and Y , (c) $P(X \geq 2 \mid Y = 1)$, (d) $E(X)$ and $E(Y)$.

(2)

Two fair dice are rolled. The sum of the outcome is denoted by X and the absolute value of the difference by Y . Calculate the joint probability mass function of X and Y and the marginal probability mass functions of X and Y .

(3)

Let the joint probability density function of random variables X and Y be given by

$$f(x, y) = \begin{cases} 2 & \text{if } 0 \leq y \leq x \leq 1 \\ 0 & \text{elsewhere.} \end{cases}$$

- (a) Calculate the marginal probability density functions of X and Y , respectively.
- (b) Find $E(X)$ and $E(Y)$.
- (c) Calculate $P(X < 1/2)$, $P(X < 2Y)$, and $P(X = Y)$.
- (d) Are X and Y independent? Why or why not?
- (e) $f_{X|Y}(x \mid y) = ?$

(4)

Let the joint probability mass function of random variables X and Y be given by

$$p(x, y) = \begin{cases} \frac{1}{7}x^2y & \text{if } (x, y) = (1, 1), (1, 2), (2, 1) \\ 0 & \text{elsewhere.} \end{cases}$$

Are X and Y independent? Why or why not?

(5)

Let the joint probability density function of X and Y be given by

$$f(x, y) = \begin{cases} 2e^{-(x+2y)} & \text{if } x \geq 0, \quad y \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

Find $E(X^2Y)$.

(6)

Let the joint probability mass function of discrete random variables X and Y be given by

$$p(x, y) = \begin{cases} \frac{1}{25}(x^2 + y^2) & \text{if } x = 1, 2, \quad y = 0, 1, 2 \\ 0 & \text{otherwise.} \end{cases}$$

Find $p_{X|Y}(x|y)$, $P(X = 2 \mid Y = 1)$, and $E(X \mid Y = 1)$.

(7)

A point (X, Y) is selected randomly from the triangle with vertices $(0, 0)$, $(0, 1)$, and $(1, 0)$.

- (a) Find the joint probability density function of X and Y .
- (b) Calculate $f_{X|Y}(x|y)$.
- (c) Evaluate $E(X \mid Y = y)$.